The Use of Features Extracted from Noisy Samples for Image Restoration Purposes

Luminiţa STATE, Cătălina COCIANU, Panayiotis VLAMOS

An important feature of neural networks is the ability they have to learn from their environment, and, through learning to improve performance in some sense. In the following we restrict the development to the problem of feature extracting unsupervised neural networks derived on the base of the biologically motivated Hebbian self-organizing principle which is conjectured to govern the natural neural assemblies and the classical principal component analysis (PCA) method used by statisticians for almost a century for multivariate data analysis and feature extraction. The research work reported in the paper aims to propose a new image reconstruction method based on the features extracted from the noise given by the principal components of the noise covariance matrix. The computation of the features of the noise $\eta \sim N(0,\Sigma)$ *is carried out by a PCA neural network trained by the GHA as well as alternative approaches as for instance Generalized Recursive Least Square (GRLS) algorithm and APEX . The training process aims the learning of the p most significant eigen-vectors of* Σ*, that is the eigenvectors corresponding to the largest p eigen-values of* Σ.
Keywords: *feature extraction, PCA, Generalized Hebbian Algorithm, image restoration, wavelet transform, multiresolution support set.*

1 General view on image restoration tasks

The effectiveness of restoration techniques mainly depends on the accuracy of the image modeling. Many image-degradation models have been developed based on different assumptions. One of the most popular degradation models is the linear continuous image-degradation where it is assumed that the image blur can be modeled as a superposition with an impulse response H that may be space variant and its output is subject to an additive noise.

The restoration can be viewed as a process that attempts to reconstruct or recover an image that has been degraded by using some *a priori* knowledge about the degradation phenomenon. Thus restoration techniques are oriented toward modeling the degradation and applying the inverse process in order to recover the original image. This approach usually involves formulating a criterion of goodness that will yield some optimal estimate of the desired result.

The inverse-filter method works only for extremely high signal-to-noise-ratio images. The Wiener filter is usually implemented only after the WSS assumption has been made for images. Furthermore, knowledge of the power spectrum or correlation matrix of the clean image is required. Usually, additional assumptions regarding boundary conditions are made so that fast orthogonal transforms can be used. Approaches based on noncausal models such as the noncausal autoregressive of Gaussian-Markov random-field models (Chellappa and Kashyap, 1982, Jinchi and Chellappa, 1985) also make assumptions such as WSS and periodic boundary conditions.

Generally speaking, the multiresolution algorithms perform the restoration tasks by combining, at each resolution level, according to a certain rule, the pixels of a binary support image. The values of the support image pixels are either 1 or 0 depending on their significance degree. At each resolution level, the contiguous areas of the support image corresponding to 1-value pixels are taken as possible objects of the image. The multiresolution support set is a data structure suitable for developing noise removal algorithms. The multiresolution algorithms perform the restoration tasks by combining, at each resolution level, according to a certain rule, the pixels of a binary support image. The values of the support image pixels are either 1 or 0 depending on their significance degree. At each resolution level, the contiguous areas of the support image corresponding to 1-value pixels are taken as possible objects of the image. The multiresolution support is the set of all support images. The multiresolution support can be computed using the statistically significant wavelet coefficients.

An important feature of neural networks is the ability they have to learn from their environment, and, through learning to improve performance in some sense. In the following we restrict the development to the problem of feature extracting unsupervised neural networks derived on the base of the biologically motivated Hebbian self-organizing principle which is conjectured to govern the natural neural assemblies and the classical principal component analysis (PCA) method used by statisticians for almost a century for multivariate data analysis and feature extraction.

The purpose of an algorithm for self-organizing learning is to discover significant patterns or features in the input data without the help provided by an external teacher. The ability to adapt to the environment without the provision of an external teacher is encountered in nature in most intelligent organisms. In this paradigm, the lack of teaching signals is compensated for by an inner purpose, i.e., some built-in criterion or objective function that the system seeks to optimize.

Typically, the purpose is twofold, the extraction of significant features of the input data, and to cluster data in neighborhoods based on a certain similarity criterion aiming the development of an image restoration technique.

In the following we restrict the development to the problem of feature extracting unsupervised neural networks derived on the base of the biologically motivated Hebbian self-organizing principle which is conjectured to govern the natural neural assemblies and the classical principal component analysis (PCA) method have been used for long in multivariate data analysis and feature extraction.

Both ends of the connection have appealing properties resulting from residing from the simplicity and locality of the Hebbian type learning and the optimality of the PCA method in dimensionality reduction. Classical PCA is based on the second-order statistics of the data and, in particular, on the eigen-structure of the data covariance matrix and accordingly, the PCA neural models incorporate only cells with linear activation functions. More recently, several generalizations of the classical PCA models to non-Gaussian models, the Independent Component Analysis (ICA) and the Blind Source Separation techniques (BSS) have become a very attractive and promising framework in developing more efficient image restoration algorithms.

2. PCA algorithms based on the hebbian learning

The input signal is modeled as a wide-sense-stationary n-dimensional process $(X(t), t \geq 0)$ of mean $E(X(t)) = 0$ and covariance matrix $E\left(X(t)X(t)^T\right) = S$. Let $\Phi_1,...,\Phi_n$ be the eigen-vectors of S taken according to the decreasing order of their corresponding eigen-values $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$. The architecture of a PCA neural network consists of the n-neuron input layer and the m-neuron computation layer. A PCA algorithm assures that the most significant features $\Phi_1,...,\Phi_m$ become asymptotically as the local memories corresponding as synaptic vectors $W_1,...,W_m$ of the neurons in the computation layer.

Let us denote by $W(t) = (W_1(t),...,W_m(t))$ the synaptic memory at the moment t, and let $Y(t) = (Y_1(t),...,Y_m(t))^T$ be the output of the computation layer where $Y_j(t) = W_j^T(t)X(t), 1 \leq j \leq m$.

The Hebbian learning rule for learning the first principal component is,

$$W_1(k+1) = W_1(k) + \eta(k)X(k)Y_1(k) \quad (2.1)$$

where the sequence of learning rates $(\eta(k))$ are taken such that the conditions of the Kushner theorem hold (Kushner and Clark, 1978).

The Equation (2.1) is not interesting because the resulting algorithm is unstable that is $\lim_{k\to\infty}\|W_1(k)\| = \infty$ but asymptotically $W_1(k)$ is directed toward $L(\Phi_1)$ the linear subspace generated by Φ_1, hence the instability is the only obstacle inhibiting Equation (2.1) from being a real principal component analyzer.

Oja and Karhunen (Karhunen and Oja, 1982, Oja, 1991 and Oja, 1992) proposed a first order approximation of the Hebbian learning rule,

$$W_1(k+1)=W_1(k)+\eta(k)\left(X(k)Y_1(k)-Y_1^2(k)W_1(k)\right) \quad (2.2)$$

The Generalized Hebbian Algorithm (GHA) (Haykin 1999) is one of the first neural models for extracting multiple PCs. The idea of GHA is to use the Hotelling deflation technique and the Oja's algorithm for learning as many principal components as are required. The GHA learning scheme is given by,

$$W_1(k+1)=W_1(k)+\eta(k)\left(X(k)Y_1(k)-Y_1^2(k)W_1(k)\right) \quad (2.3)$$

$$W_j(k+1)=W_j(k)+\eta(k)\left(\tilde{X}_j(k)\tilde{Y}_j(k)-\tilde{Y}_j^2(k)W_j(k)\right), \quad (2.4)$$

for $2\le j\le m$, where $Y_j(k)=W_j^T(k)X(k)$,

$\tilde{X}_j(k)=\tilde{X}_{j-1}(k)-Y_{j-1}(k)W_{j-1}(k)=$

$=\sum_{i=1}^{j-1}Y_i(k)W_i(k)$, $\tilde{Y}_j(k)=W_j^T(k)\tilde{X}_j(k)$.

The Adaptive Principal Component Extraction (APEX) rule was proposed by Kung and Diamantaras (Diamantaras and Kung, 1996 and Kung and Diamantaras, 1991). The model allows the extraction of multiple PCs using lateral connections between the output neurons instead of using an explicit, off-line deflation transformation.

The lateral connection from the *p*th neuron to the *j*th neuron is weighted by $a_{pj}(t)$, $2\le j\le m$, $1\le p\le j-1$.

The learning equations changes the lateral connection weights and the synaptic memories according to,

$$W_j(k+1)=W_j(k)+\eta(k)\left(\tilde{X}_j(k)Y_j(k)-Y_j^2(k)W_j(k)\right) \quad (2.5)$$

$$Y_j(k)=W_j^T(k)X(k)-\sum_{i=1}^{j-1}a_{ij}(k)Y_i(k) \quad (2.6)$$

Assuming that at the moment t_0, $W_i(t_0)\approx\Phi_i$, for all $1\le i\le j-1$, the ODE assigned to the APEX algorithm for encoding the *j*th eigen vector as the synaptic memory, according to Kushner theorem is,

$$\frac{dW_j(t)}{dt}=SW_j(t)-\sum_{l=1}^{j-1}\lambda_l\Phi_l a_{lj}(t)-\sigma_j(t)W_j(t), \quad (2.7)$$

$1\le j\le m$

$$\frac{da_{ij}(t)}{dt}=\lambda_i\left(W_j^T(t)\Phi_i-a_{ij}(t)\right)-\sigma_j(t)a_{ij}(t), \quad (2.8)$$

$2\le j\le m$, $1\le i\le j-1$, where $q_j(t)=W_j(t)-\sum_{l=1}^{j-1}a_{lj}(t)\Phi_l$, $\sigma_j(t)=q_j^T(t)Sq_j(t)$.

3. The noisy features based PCA algorithm For image restoration purposes

The research work reported in this paper aims to propose a new image reconstruction method based on the features extracted from the noise given by the principal components of the noise covariance matrix. The computation of the features of the noise $\eta\sim N(0,\Sigma)$ is carried out by a PCA neural network trained by the GHA as well as alternative approaches as for instance Generalized Recursive Least Square (GRLS) algorithm and APEX (Haykin, 1999; Kung, Diamantaras, 1990; Diamantaras, Kung, 1996). The training process aims the learning of the *p* most significant eigen-vectors of Σ, that is the eigenvectors corresponding to the largest *p* eigen-values of Σ.

We consider the additive normal distributed degradation model. Let I^0 be a RxC matrix, where representing the initial image of *L* gray levels and let I be the distorted variant resulted from I^0 by additively superimposing random noise distributed $N(0,\Sigma)$. Both images are split into blocks of size *n*, where *n* is a natural number dividing *C*, $C=nC_1$. For $\forall i=1,...,R$, $j=1,...,C_1$, we denote by $I_{i,j}$ the sequence of *n* pixels of the *i-th* row starting with the $n(j-1)$+1-th pixel and ending with the *nj*-th of the image *I*. Similarly, the block denoted by $I_{i,j}^0$ is the sequence of *n* pixels of the *i-th* row from the $n(j-1)$+1-th pixel to the *nj*-th corresponding to the image I^0. Therefore,

$\forall i=1,...,R$, $j=1,...,C_1$, $I_{i,j}=I_{i,j}^0+\eta$,

where η is a *n*-dimensional random vector distributed $N(0,\Sigma)$.

The algorithm for removing the noise component proceeds in two stages as depicted in Figure 1:

- in the first stage the noise features Φ are computed. The columns of Φ are the eigen vectors of Σ, taken according to the decreasing order of their corresponding eigen- values;
- in the second stage, using Φ, we apply a noise removal method *M* for cleaning each pixel (i,j) of the decorrelated transformed image.

$$I \xrightarrow{\Phi^T} I' = T_{\Phi^T}(I) \xrightarrow{M} I'' \xrightarrow{\Phi} \tilde{I} = T_{\Phi}(I'')$$

Fig.1.

The restoration process of the image *I* using the learned features is performed as follows:

Step 1. Compute the image *I'* by decorrelating the noise component,

$\forall i = 1,...,R$, $j = 1,...,C_1$,

$I'_{i,j} = \Phi^T I_{i,j} = \Phi^T I^0_{i,j} + \eta'$, where

$\eta' = \Phi^T \eta \sim N(0, \Sigma')$, $\Sigma' = \Phi^T \Sigma \Phi = \Lambda$,

$\Lambda = diag\{\lambda_1, \lambda_2, ..., \lambda_n\}$.

Step 2. The noise component η' is removed for each pixel *P* of the image *I'* using the multirezolution support of *I'* by the labeling method of each wavelet coefficient of P, resulting *I''*.

$I''_{i,j} = MST(I'_{i,j}) \cong \Phi^T I^0_{i,j}$, $\forall i = 1,...,R$,

$j = 1,...,C_1$,

where $MST(I'_{i,j})$ is produced by applying the above mentioned method to $I'_{i,j}$.

Step 3. An approximation $\tilde{I} \cong I^0$ of the initial image I^0 is produced by applying the inverse transform of T_{Φ^T} to *I''*,

$\tilde{I}_{i,j} = \Phi I''_{i,j} \cong \Phi \Phi^T I^0_{i,j} = I^0_{i,j}$, $\forall i = 1,...,R$,

$j = 1,...,C_1$

Note that the decorrelation of the noise component is performed by the computation carried out at **Step 1** because the resulted image is

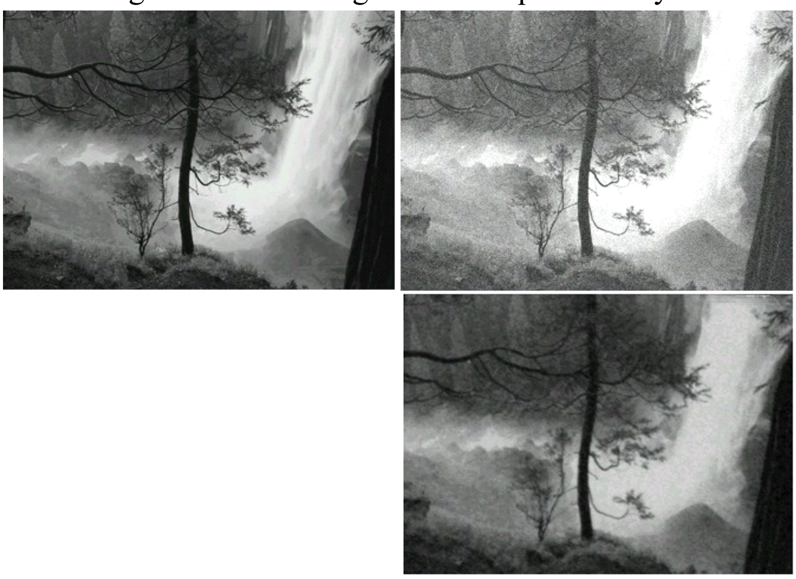
$I'_{i,j}(k) = \Phi^T I^0_{i,j}(k) + \eta'(k)$,

$k = n(j-1),...,nj$, $j = 1,...,C_1$,

where for each $k = n(j-1),...,nj$, $j = 1,...,C_1$,

$\eta'(k) \sim N(0, \sigma^2_{i,k})$, $\sigma^2_{i,k} = \lambda_{k,k}$.

The original “clean” image A sample of noisy variant



The restored image

Fig. 2.

4. Experimental results

Our tests were performed on monochrome images distorted by $N(m,\sigma^2)$ - distributed noise. The learning process encoded the principal components in about 2000 steps, the quality of the restored image being “enough good” for the approximations resulted so far. In the sequel, we present some samples represented in figures 2 and 3, where $m = 50$ and $\sigma^2 = 100$.

The original “clean” image A sample of noisy variant

The restored image

Fig. 3.

5. Conclusions and suggestions for further work

The good results obtained in using the above presented algorithm encourage the research in improving it in several ways. On one hand, improvement is expected by including some sort of heuristic information about the class of processed images, when it is available and also by using higher order statistics on the wavelet coefficients. On the other hand, extensions of it in a ICA and BSS frameworks are expected to supply better results concerning the quality of the restored images.

References

[1] Chatterjee, C., Roychowdhury, V.P. & Chong E.K.P. (1998). On Relative Convergence Properties of Principal Component Analysis Algorithms, IEEE Transaction on Neural Networks, vol.9,no.2

[2] Chellappa, R. & Jinchi, H. (1985). A nonrecursive Filter for Edge Preserving Image Restoration, In: Proc. Intl.Conf.on Acoustic, Speech and Signal Processing, Tampa

[3] Chellappa, R. & Kashyap,R.L. (1982). Digital Image Restauration Using Spatial Interaction Models, In: Proc. Intl.Conf.on Acoustic, Speech and Signal Processing, vol. ASSP-30

[4] Cocianu, C., State, L., Ştefănescu, V. & Vlamos P. (2004). On the Efficiency of a Certain Class of Noise Removal Algorithms in Solving Image Processing Task, *Proceedings of 1st International Conference on Informatics in Control, Automation and Robotics, ICINCO 2004, August 25-28, Setubal, Portugal*, INSTICC Press, pp. 320-323, 2004,

[5] Cocianu, C., State, L., Ştefănescu, V. & Vlamos P. (2003). Noise Removal Techniques Using the Multiresolution Representation, In: Proceedings International Conference on Computer&Industrial Engineering (ICC&IE), Limerick, Ireland, August 11th-13th

[6] Diamantaras, K.I. & Kung, S.Y. (1996), Principal Component Neural Networks: theory and applications, John Wiley &Sons
[7] Haykin, S. (1999), Neural Networks A Comprehensive Foundation, Prentice Hall,Inc.
[8] Hastie, T., Tibshirani, R., Friedman, J. (2001) *The Elements of Statistical Learning*, Springer
[9] Hyvarinen, A., & Hoyer, P., & Oja, E. (1999). Image Denoising by Sparse Code Shrinkage, www.cis.hut.fi/projects/ica, November, 1999
[10] Karhunen, J. &. Oja, E. (1982). New Methods for Stochastic Approximations of Truncated Karhunen-Loeve Expansions, In: Proceedings 6th International Conference on Pattern Recognition, Springer Verlag
[11] Kung, S.Y. &. Diamantaras, K.I. (1990). A Neural Network Learning Algorithm for Adaptive Principal Component Extraction (APEX), In: IEEE Transaction on Acoustics, Speech and Signal Processing
[12] Kushner, H.J. & Clark, D.S. (1978). Stochastic Approximation Methods for Constrained and Unconstrained Systems, Springer Verlag
[13] Mao, J. & Jain,A.K. (1995). Artificial Neural Networks for Feature Extraction and Multivariate Data Projection, In: IEEE Transaction on Neural Networks, vol.6, no.2, 1995
[14] Matsuoka, K. & Kawamoto, M. (1994). A Neural Network that Self-Organizes to Perform Three Operations Related to Principal Component Analysis, Neural Networks, vol.7, no.5, 1994
[15] Oja, E. (1991). Data Compression, Feature Extraction and Autoassociation in Feedforward Neural Networks, vol. 1, North Holland
[16] Oja, E. (1992). Principal Components, Minor Components and Linear Neural Networks, Neural Networks, vol. 5
[17] Sonka, M. & Hlavac, V., (1997). Image Processing, Analyses and Machine Vision, Chapman & Hall Computing
[18] State, L, Cocianu, C. & Vlamos, P. (2001). Attempts in Using Statistical Tools for Image Restoration Purposes, In: Proceedings of SCI2001, Orlando, USA, July 22-25
[19] State, L, Cocianu, C. & Vlamos, P. (2001). A Regressive Technique of Image Restorations, In: Proceedings of the 29th ICC&IE, Nov. 1-3 Montreal, Canada
[20] State, L, Cocianu, C., Stefanescu, V. & Vlamos, P. (2003). A Comparative Analysis on the Performance of a certain class of PCA Algorithms in solving Image Processing Tasks, In: Proceedings of the 7th World Multiconference on Systemics, Cybernetics and Informatics (SCI 2003), Orlando, USA, July 27-30
[21] Umbaugh, S. (1998). Computer Vision and Image Processing, Prentice Hall